Group Theory and Applications

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Monday talks

Clifford-Fischer Theory Applied to $\overline{G_1} = 2^{4\cdot}A_8$

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Based on joint work with **Chrisper Chileshe and Jamshid Moori**.

In this talk, we demonstrate how to compute the character table of a non-split group extension $\overline{G_1} = 2^{4\cdot}A_8$ using a method known as Clifford-Fischer theory. The character tables of all the maximal subgroups of the sporadic simple groups are known, except for some maximal subgroups of the Monster M and Baby Monster B. Unfortunately, the character table of a non-split extension group is not easy to compute since all irreducible characters of inertia factor groups are not extendable. Nevertheless, a number of authors [1], [2], [3], have worked on non-split extensions for instance, Ali [1], among others considered the projective representations and characters and showed how the technique of coset analysis and Clifford-Fischer theory can be applied to non-split extensions $3^{7\cdot}O_7(3)$ and $3^{7\cdot}O_7(3)$:2, which are maximal subgroups of Fischer's largest sporadic simple group Fi'_{24} and its automorphism group Fi_{24} respectively. This is our motivation in this talk as we show how to compute the character table of $\overline{G_1}$.

Reference

[1] F. Ali, Fischer-Clifford Theory for Split and Non-Split Group Extensions, PhD Thesis, University of Natal, Pietermaritzburg, 2001

[2] A.B.M. Basheer and J. Moori, Clifford-Fischer Theory Applied to Certain Groups Associated with Symplectic, Unitary and Thompson Groups, PhD Thesis, University of Kwazulu-Natal, Pietermaritzburg, 2012.

[3] T.T. Seretlo, Fischer-Clifford Matrices and Character Tables of Certain Groups Associated with Simple Groups $O_8^+(2)$, HS and Ly, PhD Thesis, University of Kwazulu-Natal, Pietermaritzburg, 2012.

Average number of zeros of characters of finite groups.

Yash Madanha

University of Pretoria

Dedicated to the memory of **Kay Magaard**.

There has been some interest on how the average character degree affects the structure of a finite group. We define, and denote by $\mathrm{anz}(G)$, the average number of zeros of characters of a finite group G as the number of zeros in the character table of G divided by the number of irreducible characters of G. We show that if $\mathrm{anz}(G) < 1$, then the group G is solvable and also that if $\mathrm{anz}(G) < \frac{1}{2}$, then G is supersolvable. We characterise abelian groups by showing that $\mathrm{anz}(G) < \frac{1}{3}$ if and only if G is abelian.

On some triply-even binary codes invariant under $\mathrm{PSL}_2(p^2)$

Xavier Mbaale

School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal and Department of Mathematics, University of Zambia

Based on joint work with **Bernardo Rodrigues**.

Using an interplay between combinatorial designs and finite geometries, a geometric structure known as a finite inversive plane is studied in connection with a symmetric 1-$(\frac{p(p^2+1)}{2}, p^2-1, p^2-1)$ design $\mathscr{D}$ invariant under the group $\mathrm{PSL}_2(p^2)$. The design $\mathscr{D}$ is defined by the primitive action of the projective special linear groups $\mathrm{PSL}_2(p^2)$, where p is an odd prime, on the circles (points) of the finite miquelian inversive planes. It is shown that $\mathscr{D}$ is quasi-symmetric with intersection numbers $\{2(p-1), 2(p+1)\}$. Further, from the row span of the incidence matrix of $\mathscr{D}$, we construct an infinite family of triply-even binary codes and show that $\mathscr{C}$ is invariant under the group $P\Sigma L(2, p^2)$.

Reference

[1] E. F. Assmus, Jr and J. D. Key. *Designs and their Codes*. Cambridge: Cambridge University Press, 1992. Cambridge Tracts in Mathematics, Vol. 103 (Second printing with corrections, 1993).

[2] K. Betsumiya and A. Munemasa. On triply even binary codes. *J. Lond. Math. Soc*, **86** (1) (2012) 1–16.

[3] O. H. King. The subgroup structure of finite classical groups in terms of geometric configurations . In: *B.S.Webb, ed. Survey in Combinatorics 2005, 29—56,* London Math. Soc. Lecture Note Ser., **327**, Cambridge Univ. Press, Cambridge, 2005.

[4] X. Mbaale and B. G. Rodrigues. Symmetric 1-designs from $\mathrm{PSL}_2(q)$, for q a power of an odd prime. *Transactions on Combinatorics,* to appear.

On a Maximal Subgroup of The Orthogonal Group $O_8^+(3)$

David Musyoka

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Based on joint work with **Lydia Njuguna** (Kenyatta University), **Lucy Chikamai** (Kibabii University) and **Abraham Prins** (Nelson Mandela University-SA)

The orthogonal group $O_8^+(3)$ of order $4952178914400 = 2^{12}.3^{12}.3^2.7.13$ has 27 conjugacy classes of maximal subgroups [2]. Among the maximal subgroups of $O_8^+(3)$ are three non-conjugate but isomorphic subgroups $\bar{G}_1$, $\bar{G}_2$ and $\bar{G}_3$ of the form $3^6{:}L_4(3)$ order 4421589120 and index 1120 in $O_8^+(3)$. The aim of this study is to compute the Fischer-Clifford matrices and hence the character table of $\bar{G}_1$, the first group of the three as they appear in ATLAS of finite groups [2]. For this purpose we use the Fischer-Clifford matrices technique which is based on Clifford theory and was developed by Bernd Fischer [3]. This technique relies on the fact that every irreducible character of an extension group $\bar{G} = N : G$ can be obtained by induction from the inertia groups of $\bar{G}$. The method of coset analysis which was developed and first used by Moori [4] has been used largely to determine the conjugacy classes of extensions of elementary abelian groups. The group $3^6{:}L_4(3)$ which we shall now denote by $\bar{G}$ is a split-extension of $N = 3^6$, the vector space of dimension 6 over $GF(3)$ by the projective special linear group $G = PSL_4(3)$ also denoted as $L_4(3)$. In this presentation, we present our results on this study by briefly describing the action of $L_4(3)$ on $N = 3^6$, conjugacy classes of the group $\bar{G}$, identification of Inertia factor groups, computation of unique Fischer matrices and fusion of $\bar{G}$ into $O_8^+(3)$. Most of our computations are carried out using computer algebra systems MAGMA [5] and GAP [1].

Reference

[1] The GAP Group, GAP Groups,Algorithms, and Programming, Version GAP 4.11.0 of 29-Feb-2020, https://www.gap-system.org

[2] J. H. Conway, Atlas of finite groups: maximal subgroups and ordinary characters of simple groups, Oxford University Press, 1985.

[3] B. Fischer, Clifford-Matrices, in: Representation Theory of Finite Groups and Finite Dimensional Algebras, Springer, 1991, pp. 1-16.

[4] J. Moori, On the groups G^+ and $\bar{G}$ of the forms $2^{10} : M_{22}$ and $2^{10} : \bar{M}_{22}$, Journal PhD thesis, University of Birmingham, year 1975.

[5] W. Bosma, J. Cannon, C. Playoust, The magma algebra system (1996).

On some binary codes of lenght 120 invariant under A_9

Cedric Ndarinyo

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Based on joint work with **Lucy Chikamai**.

Let G be the alternating group A_9. We determine all binary codes constructed from the primitive permutation representation of G of degree 120. We investigate the properties of these codes especially those of small dimension. We establish that there is no self-dual code of length 120 invariant under A_9. We also determined a strongly regular graph with parameters $(256, 135, 70, 72)$ and several designs whose automorphism group is G.

On a GAP routine for projective characters of a finite group

Abraham Love Prins

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It is a well-known fact that all the sets of irreducible projective characters $\mathrm{IrrProj}(G, \alpha_i)$, i=$1, 2, \ldots, m$, of a finite group G with factor sets α_i can be obtained from the ordinary irreducible characters of a so-called representation group $R \cong M(G).G$ of G, where $M(G)$ denotes the Schur multiplier of the group G and m the number of cohomology classes $[\alpha_i]$ in $M(G)$. Using this theory, a routine written in the computational algebra system GAP is presented to compute the sets $\mathrm{IrrProj}(G, \alpha_i)$ for a finite group G. In particular, this said GAP routine will be applied to the maximal subgroups of the sporadic simple Mathieu group M_{22}.

Linear codes of the Mathieu groups and their support designs

Amin Saeidi

University of Tehran

In Memory of Dr. Karim Ahmadidelir, 1964-2020

Based on joint work with **Mohammad Darafsheh and Bernardo Rodrigues**.

In this talk, using a representation theoretic method we obtain all binary linear codes that admit the Mathieu group M_{11} as a primitive permutation automorphism group. We also construct some point- and block primitive 1-designs from the supports of these codes, including a 3-$(12, 6, 10)$ design and a Steiner system $S(9, 9, 12)$. We use the triangular graphs to compute the stabilizers of the codes and define some new graphs to study the structure of the codes in general. We may generalize these methods to other suitable families of finite simple groups and obtain similar results.

Reference

[1] E. F. Assmus, Jr and J. D. Key. *Designs and their Codes*. Cambridge: Cambridge University Press, 1992. Cambridge Tracts in Mathematics, Vol. 103 (Second printing with corrections, 1993).

[2] W.H. Haemers, R. Peeters, J.M. van Rijckevorsel, *Binary codes of strongly regular graphs,* Des. Codes Cryptogr. 17 (1999) 187–209.

[3] W. Knapp and P. Schmid, Codes with prescribed permutation automorphism. *J. Algebra* **67** (2) (1980), 415–435.

Which groups can act on algebras of Monster type?

Sergey Shpectorov

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Based on joint work with **Clara Franchi and Mario Mainardis**.

Axial algebras are a class of non-associative algebras closely related to groups. Much of recent research in this area is focussed on the subclass of algebras of Monster type. The interest in this particular type of axial algebras is due to the known examples, including the Jordan algebras for classical and some exceptional groups, Matsuo algebras for 3-transposition groups, and the Griess algebra and its subalgebras, corresponding to the Monster sporadic simple group M and various subgroups of it. Recently a concept of a double axis and of a flip subalgebra were introduced leading to a rich variety of new examples.

We will focus on the following question: what are the properties that are possessed by the groups arising from axial algebras of Monster type? It is well known that the Monster M is a 6-transposition group. A theorem of Sakuma, proved initially in the context of vertex operator algebras, gives an explanation to this fact. More recently, Sakuma's theorem has been transferred into the context of axial algebras and broadly generalized. In particular, we have been able to completely classify the generic case, that is, the algebras that exist over all fields. It turns out that the groups coming from the generic algebras are groups of 3-transpositions and hence they are all known. This opens up the possibility of a complete classification of generic algebras in terms of their groups.

In the lecture, after reviewing the basics of axial algebras, we will discuss these and other related results and pose a few open problems.

Axial algebras for the sporadic simple group HS

Tendai M Mudziiri Shumba

University of Johannesburg

We present constructions of axial algebras for the Higman-Sims sporadic simple group via Norton algebras. Fusion laws are presented as well as the extensions of these algebras by unit.

Groups acting with fixity at most 4

Rebbeca Waldecker

Institute of Mathematics, Martin Luther University Halle-Wittenberg

In this talk we will see how the theory of Riemann surfaces leads to interesting and difficult questions about permutation groups and, ultimately, about finite simple groups. We will discuss the motivation, methods, some results and open problems.

Generating graphs with clique number 3 and just coverable groups

Bettina Wilkens

Department of Mathematics, University of Namibia

The results presented in this talk originate from a question raised by *Ekaterina Shul'man* concerning subgroup coverings of groups:
Given a covering of a group with proper subgroups taken from a set $\mathscr{S}$ and a finite n such that every element of G is in all but at most n subgroups in $\mathscr{S}$, find a bound of $|\mathscr{S}|$ in terms of n.
The -short- solution to this problem has led to a series of interconnected questions, two of which will be discussed:
Let G be a finite-two generated group. The generating graph of G has as its vertices the elements of G, an edge being placed between x and y if $\langle x, y\rangle = G$. Questions on connectedness and clique number of this graph have attracted much attention in recent years; we will present some results towards a classification of groups whose generating graph has clique number three.
Finally, we move on to finite groups G that can be covered by proper subgroups, but just barely in the sense that for every subgroup covering $\mathscr{S}$ of G there is an $x \in G$ with the property that x belongs to exactly one subgroup in $\mathscr{S}$; we present various results towards a characterisation of the nilpotent and solvable groups with this property.

Reference

[1] N. Blackburn, L. Héthelyi, *Some further properties of Soft subgroups* , Arch. Math. 69 (1997) 365 –371

[2] E. Shulman, *Subadditive set-functions on semigroups, applications to group representations and functional equations*, J. Functional Analysis 263 (2012) 1468–1484

Twisted group algebras

Wolfgang Willems

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Based on joint work with **Javier de la Cruz**.

There are some famous codes which do not occur as ideals in group algebras. For instance λ-constacyclic codes, some Hamming codes or the ternary extended $[12,6,6]$ Golay code. It turned out that all of them are ideals in a twisted group algebra. I may explain what a twisted group algebra is and explain how the above codes arise as ideals.

Designs from maximal subgroups and conjugacy classes of Ree groups

Seiran Zandi

University of KwaZulu-Natal

Based on joint work with **Jamshid Moori, Amin Saeidi and Bernardo Rodrigues**.

In this talk, our aim is to construct designs from the maximal subgroups and the conjugacy classes of the family of small Ree group ${}^2G_2(q)$, where q is an odd power of 3. The method that we use is one of two methods introduced by Key and Moori in [1, 2]. The second method introduced in [2] and called Method 2 for short, outlines the construction of 1-designs which are not necessarily symmetric. The construction of designs using Method 2 relies on a choice of a maximal subgroup M of a finite simple group G and a conjugacy class in G of some element $x \in M$.

Reference

[1] J.D. Key, J. Moori. Designs, codes and graphs from the Janko groups J_1 and J_2. *J. Combin. Math. Combin. Comput.*, **40** (2002), 143–159.

[2] J.D. Key, J. Moori. Designs from maximal subgroups and conjugacy classes of finite simple groups. *J. Combin. Math. Combin. Comput.*, **99** (2016), 41–60.
